Accelerating Branes and the String/Black Hole Transition

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String theory in Euclidean flat space with a spacelike linear dilaton contains a $D1$-brane which looks like a semi-infinite hairpin. In addition to its curved shape, this "hairpin brane" has a condensate of the open string tachyon stretched between its two sides. The tachyon smears the brane and shifts the location of its tip. The Minkowski continuation of the hairpin brane describes a $D0$-brane freely falling in a linear dilaton background. Effects that in Euclidean space are attributed to the tachyon condensate, give rise in the Minkowski case to a stringy smearing of the trajectory of the D-brane by an amount that grows as its acceleration increases. When the Unruh temperature of the brane reaches the Hagedorn temperature of perturbative string theory in the throat, the rolling D-brane state becomes non-normalizable. We propose that black holes in string theory exhibit similar properties. The Euclidean black hole solution has a condensate of a tachyon winding around Euclidean time. The Minkowski manifestation of this condensate is a smearing of the geometry in a layer around the horizon. As the Hawking temperature, T_{bh}, increases, the width of this layer grows. When T_{bh} reaches the Hagedorn temperature, the size of this "smeared horizon" diverges, and the black hole becomes non-normalizable. This provides a new point of view on the string/black hole transition.

1. Introduction

Linear dilaton backgrounds arise naturally in string theory as near-horizon (or throat) geometries of Neveu-Schwarz fivebranes and other singularities [1,2]. In the last year there has been some work on the real time dynamics of D-branes in such backgrounds. The fact that the string coupling increases as one approaches the singularity leads to an attractive potential for any D-brane localized in the throat, which causes it to roll into the strong coupling region.

The resulting trajectory was analyzed in [3] by using the Dirac-Born-Infeld (DBI) action for the D-brane. It was also pointed out in [3] that the exact boundary state describing the rolling D-brane is a Wick rotated version of the "hairpin brane" constructed previously in [4], or of its generalization to the superstring. This generalization was subsequently studied in [5-7]. Some additional interesting related work appeared in [8-29].

As discussed in [4], the hairpin brane is an open string analog of the cigar CFT, $SL(2,\mathbb{R})/U(1)$. More generally, the "paperclip" model [4], whose UV limit is the hairpin brane, is a boundary analog of the "sausage" model [30], whose UV limit is the cigar. The bulk counterpart of the Wick rotation that takes the hairpin to the rolling D-brane, takes the cigar (or Euclidean two dimensional black hole) to the Minkowski two dimensional black hole. Thus, some aspects of the dynamics of the black hole have open string counterparts. One of the motivations for the present work is to use the rolling D-brane system to learn about black holes.

For the bulk case, it is known that while for small dilaton slope the Euclidean $SL(2,\mathbb{R})/U(1)$ conformal field theory is well described by a sigma model on the cigar, for large slope a better description is in terms of Sine-Liouville CFT in the bosonic case [31,32], or $N=2$ Liouville theory in the worldsheet supersymmetric one [33]. In this description, the cigar is replaced by an infinite cylinder, and strings are prevented from exploring the strong coupling region by a condensate of a closed string tachyon which winds around the compact cycle of the cylinder and increases towards the strong coupling region.

The cigar-Liouville duality is a strong – weak coupling duality on the worldsheet. For small dilaton gradient, the sigma model on the cigar is weakly coupled since the curvature and dilaton gradient are small. For large gradient it is strongly coupled, but in that regime the Liouville description is weakly coupled, since the Liouville potential is slowly varying. In general, one has to include both the curved metric and the condensate of the winding

tachyon to get a full description of the physics. Evidence for this picture comes from both worldsheet [34,35] and spacetime Little String Theory [36,37] considerations.

The close relation between the cigar and its boundary counterpart, the hairpin, suggests [4] that a similar strong-weak coupling duality exists for the hairpin as well. In this case the statement is that for small dilaton gradient the boundary state corresponds to a brane whose shape is described by the hairpin, while for large gradient it corresponds to a brane and anti-brane at a small distance determined by the gradient, with a condensate of the open string tachyon stretched between them. We will describe this duality in more detail below.

The fact that the cigar and hairpin CFT's contain a condensate of a winding tachyon leads to an interesting question regarding the Minkowski continuations of these solutions. From the point of view of these continuations, the tachyon has non-zero winding in the Euclidean time direction, and it is not clear how to interpret it after continuation to Minkowski space. This is especially puzzling since in Euclidean space the tachyon condensate has a large effect when the dilaton gradient is of order one. In particular, above a critical value of the gradient, the hairpin and cigar states become non-normalizable as a consequence of its existence.

The purpose of this paper is to clarify this issue. Our discussion has two parts. In the first (sections 2, 3) we consider the hairpin brane and its Minkowski continuation. We identify the effects due to the winding tachyon condensate in both Euclidean and Minkowski space, and study these effects as a function of the gradient of the dilaton. In Euclidean space, the tachyon smears the hairpin shape by an amount whose size is determined by the region in which the mass squared of the tachyon is negative. The size of this region grows with the gradient of the dilaton, and diverges when the asymptotic mass of the tachyon goes to zero. At that point, the brane ceases to be normalizable.

The Minkowski continuation of the hairpin brane corresponds to a D-brane freely falling in the linear dilaton background. The smearing of the hairpin brane that is due to the condensate of the stretched tachyon gives rise after Wick rotation to a stringy smearing of the trajectory of the rolling D-brane. The size of this smearing grows with the acceleration of the D-brane. It diverges when the Unruh temperature of the brane approaches the perturbative Hagedorn temperature in the linear dilaton throat. At that point the rolling D-brane boundary state is pushed out to $\phi \rightarrow \infty$ and ceases to be normalizable.

We propose the following qualitative picture to explain this smearing. In perturbative string theory, the excitations of the D-brane are described by open strings both of whose ends lie on the D-brane. When the brane is at rest or moving at a constant velocity, its ground state has no open strings excited, and the location of the brane can be sharply defined. In our case, the continuation from Euclidean space describes a state of the accelerating brane in which there are open strings which oscillate along the ϕ axis, providing a kind of stringy halo to the brane.

The acceleration in the negative ϕ direction gives rise to a force on these strings in the opposite, positive ϕ, direction. This shifts the halo, on average, to a larger value of ϕ than the original D-brane. The size of this halo increases with the acceleration. When the Unruh temperature of the brane reaches the Hagedorn temperature, the halo extends all the way to $\phi \to \infty$ and pushes the brane out of the linear dilaton throat.

The second part of our discussion (section 4) concerns black holes. For the two dimensional $SL(2, \mathbb{R})/U(1)$ black hole we argue that in Euclidean space the winding tachyon gives rise to stringy corrections which smear the geometry of the cigar. The smearing is again determined by the region in which the mass squared of the wrapped tachyon is negative.

After Wick rotation, this results in a smearing of the region near the horizon of the Minkowski black hole. The smeared region is characterized by the fact that the local Hawking temperature in it exceeds the Hagedorn temperature. For small dilaton gradient, the size of this region is of order one in string units. One can think of it as a kind of stringy stretched horizon [38]. As the dilaton gradient increases, the size of the smeared region grows. When the Hawking temperature at infinity approaches the perturbative Hagedorn temperature in the throat, this size diverges and the black hole becomes non-normalizable.

Like in the case of the accelerating D-brane, we argue that the reason for the appearance of the stretched horizon in the Lorentzian black hole solution is a condensate of closed strings near the horizon. These strings are the Lorentzian manifestation of the winding tachyon condensate. When the Hawking temperature is close to the Hagedorn temperature, the winding tachyon is light everywhere, and in Minkowski space the closed strings that form the stretched horizon are highly excited.

We propose that the winding tachyon condensate is a general property of classical black hole solutions in string theory. For example, in the d dimensional Euclidean Schwarzschild solution, the asymptotic radius of Euclidean time (or inverse Hawking temperature) grows with the mass of the black hole M. The boundary conditions are such that a tachyon

winding once around Euclidean time survives the GSO projection, but its mass, $m_\infty(M)$ is large in this limit. Therefore, the tachyon condensate goes rapidly to zero at large distances, $T(r) \sim \exp(-m_\infty r)$. However, in a small layer near the horizon, which is again characterized by the fact that in it the local Hawking temperature exceeds the Hagedorn temperature, the tachyon condensate is appreciable, and provides some stringy smearing of the solution, as in [38]. As the mass of the black hole decreases, the width of this stretched horizon grows until, when the Hawking temperature of the black hole approaches the Hagedorn temperature, it extends all the way to infinity, and the black hole becomes non-normalizable.

The above discussion also sheds light on the string/black hole correspondence of [38,39]. The conventional picture is that the transition from black holes to strings happens when the curvature near the horizon becomes of order the string scale. A more precise formulation suggested by our considerations is that the transition takes place when the Hawking temperature of the black hole becomes equal to the perturbative Hagedorn temperature of fundamental strings in the spacetime without the black hole.

Near the transition, the black hole typically has horizon size of order l_s, but its stretched horizon has a much larger size, and as the Hawking temperature approaches the Hagedorn temperature, it looks more and more like a highly excited string state. This explains why, at the transition, its entropy must agree with that of a perturbative string state with the same mass. In order to find the value of the mass at which the transition takes place, one needs to know the exact relation between the mass and the Hawking temperature in this regime, but if one can construct an exact classical black hole solution in string theory, this formulation gives precise predictions that can in principle be verified.

2. Hairpin

In this section we describe some features of the hairpin boundary state, and its dual description in terms of boundary $N = 2$ Liouville. We restrict the discussion to the worldsheet supersymmetric case (the boundary analog of [33]) since this is the case relevant for the superstring. We will not discuss the bosonic version of this construction (the boundary analog of [31,32]).

We start with a flat Euclidean two dimensional space labeled by (ϕ, x),

$$\mathbb{R}_\phi \times \mathbb{R}_x \ . \tag{2.1}$$

The dilaton depends linearly on ϕ,

$$\Phi = -\frac{Q}{2}\phi \ . \tag{2.2}$$

We set $\alpha' = 2$, such that the central charge of ϕ is $c_\phi = 1 + 3Q^2$. Since we are planning to embed (2.1) in the superstring, there are also two free fermions, (ψ_ϕ, ψ_x) and their counterparts with the other worldsheet chirality.

The shape of the hairpin brane is described by the relation

$$e^{-\frac{Q}{2}\phi} = 2C \cos \frac{Q}{2}x \ . \tag{2.3}$$

The constant C can be set to one by shifting ϕ, and we will often do so below. As explained in [3,4], (2.3) is obtained by solving the equations of motion of the DBI action for a $D1$-brane in the linear dilaton background (2.1). This action follows from the one loop β-function of the boundary RG. Apriori, one might expect that it is accurate only for small Q and receives α' corrections. For the bosonic analog of this construction this is indeed the case [4], but for the fermionic one (2.3) is expected to be exact, in a sense that will be made more precise below.

One of the reasons for this expectation is the analogy with the bulk problem. As mentioned in the introduction, (2.3) can be thought of as an open string analog of the Euclidean cigar CFT, $SL(2, \mathbb{R})/U(1)$. In that case, it is known that while in the bosonic string the geometry receives α' corrections [40], in the fermionic string it is one loop exact [41,42] since the background preserves worldsheet $(2, 2)$ superconformal symmetry. The hairpin brane also preserves $N = 2$ superconformal symmetry on the worldsheet and is closely related to the cigar. It is natural to expect that in this case too, (2.3) is exact.

The hairpin shape (2.3) has the following properties. As $\phi \to \infty$ (*i.e.* as we approach the boundary of the linear dilaton space (2.1)), $x \to \pm\frac{\pi}{Q}$. In this region the system looks like a $D1$-brane and an anti $D1$-brane separated by the distance $\delta x = \frac{2\pi}{Q}$. As we move to smaller ϕ, the two D-branes curve towards each other as follows:

$$\delta x = \frac{2\pi}{Q} - \lambda e^{-\frac{Q}{2}\phi} + O(e^{-Q\phi}) \ , \tag{2.4}$$

where λ is a positive constant. They meet at $x = 0$, where the string coupling takes its largest value along the brane. Since we can choose this value to be arbitrarily small, we expect the boundary CFT describing the hairpin brane to be well behaved on the disk.

An important probe of the hairpin brane is the disk one point function of the bulk operators

$$T(p, q) = e^{(-\frac{Q}{2} + ip)\phi + iqx} \ . \tag{2.5}$$

These operators have worldsheet scaling dimension $\Delta_{p,q} = \bar{\Delta}_{p,q} = \frac{1}{2}p^2 + \frac{1}{2}q^2 + \frac{1}{8}Q^2$. Their one point function is given by [4,5]

$$\Psi(p, q) = \langle T(p, q) \rangle \simeq \frac{\Gamma(1 - iQp)\Gamma(-i\frac{2p}{Q})}{\Gamma(\frac{1}{2} - i\frac{p}{Q} + \frac{q}{Q})\Gamma(\frac{1}{2} - i\frac{p}{Q} - \frac{q}{Q})} \tag{2.6}$$

where we neglected an overall constant that will not play a role below.

The one point function (2.6) determines the coefficient of the identity in the hairpin boundary state:

$$|\text{hairpin}\rangle = \Psi(\phi, x)|0\rangle + \cdots \tag{2.7}$$

where the $\cdots$ stands for oscillator states, and

$$\Psi(\phi, x) = \int \frac{dp dq}{(2\pi)^2} e^{-ip\phi - iqx} \Psi(p, q) \tag{2.8}$$

is the position space wave function of the hairpin.

In the limit $Q \to 0$, the wave function $\Psi(p, q)$ (2.6) can be approximated by

$$\Psi_{\text{cl}}(p, q) \simeq \frac{\Gamma(-i\frac{2p}{Q})}{\Gamma(\frac{1}{2} - i\frac{p}{Q} + \frac{q}{Q})\Gamma(\frac{1}{2} - i\frac{p}{Q} - \frac{q}{Q})} \ . \tag{2.9}$$

Using this classical wave function in (2.8), gives [5]

$$\Psi_{\text{cl}}(\phi, x) = \int \frac{dp dq}{(2\pi)^2} e^{-ip\phi - iqx} \Psi_{\text{cl}}(p, q) \simeq \delta\left(\phi + \frac{2}{Q} \ln\left[2\cos\left(\frac{Q}{2}x\right)\right]\right) \ , \tag{2.10}$$

which is localized on the classical hairpin (2.3).

In the opposite limit,[1] $Q \to \infty$, the wave function (2.6) goes like (again, neglecting an overall constant)

$$\widetilde{\Psi}(p, q) \simeq \Gamma(-iQp) \cos \pi \frac{q}{Q} \ , \tag{2.11}$$

and the position space wave function (2.8) behaves like

$$\widetilde{\Psi}(\phi, x) \simeq e^{-e^{-\frac{1}{Q}\phi}} \left[\delta(x - \frac{\pi}{Q}) + \delta(x + \frac{\pi}{Q})\right] \ . \tag{2.12}$$

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[1] In string theory applications Q is actually bounded from above, $Q \leq 2$, but here we are discussing boundary CFT where this can be ignored.

In this limit we have two branes localized at $x = \pm\frac{\pi}{Q}$ and a potential preventing the system from exploring the strong coupling region $\phi \to -\infty$.

A more precise description of the hairpin in the limit $Q \to \infty$ is the following. We have a $D1$-brane and an anti $D1$-brane at a distance

$$\delta x = \frac{2\pi}{Q} \ . \tag{2.13}$$

Despite the fact that the brane and anti-brane are very close, there is no instability associated with their attraction, since for large Q the translational mode of the branes in the x direction is very heavy. A condensate of the open string tachyon stretched between the two branes generates a boundary perturbation of the form[2]

$$\delta S_{ws} = \tilde{\lambda} \int d\tau d\theta e^{-\frac{1}{2Q}(\phi+i\tilde{x})} + \text{c.c.} \ , \tag{2.14}$$

where $\tilde{x}$ is the T-dual of x, τ is the coordinate along the boundary of the worldsheet, and θ is its worldsheet superpartner. Some comments:

(1) The interaction (2.14) is the boundary $N = 2$ Liouville superpotential

$$W(\Phi) = \tilde{\lambda}e^{-\frac{1}{2Q}\Phi} \ , \qquad \Phi = \phi + i\tilde{x} \ . \tag{2.15}$$

It is a boundary analog of the bulk $N = 2$ Liouville superpotential of [33].

(2) The boundary superpotential (2.15) is non-normalizable when

$$-\frac{1}{2Q} > -\frac{Q}{4} \ , \tag{2.16}$$

i.e. for $Q^2 > 2$, precisely like its bulk counterpart.

(3) The superpotential (2.15) leads to (2.12), using the fact that the corresponding potential behaves like $V = |W|^2 \simeq \exp(-\frac{1}{Q}\phi)$.

So far we discussed the properties of the hairpin boundary state only in the limit of very small and very large Q. For finite Q one needs to take into account both the shape of the brane (2.3), and the stretched tachyon condensate (2.14). As a concrete application of this, consider the analytic structure of the one point function (2.6). As is familiar from the cigar/Liouville correspondence (see e.g. [32]) the poles of the one point function are due

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[2] As usual for boundary tachyon perturbations, one has to add a boundary fermionic degree of freedom which also appears in the boundary perturbation. We will omit it here.

to processes that can occur anywhere in the bulk of $\mathbb{R}_\phi$ and can be studied by expanding in the interactions.

One set of poles occurs when iQp is a positive integer, and can be understood by expanding the one point function in powers of the boundary $N = 2$ Liouville superpotential (2.14). Winding conservation in the bulk of $\mathbb{R}_\phi$ implies that non-zero contributions go like $|\tilde{\lambda}|^{2n}$ with n a positive integer. Anomalous conservation of ϕ momentum implies that

$$-\frac{Q}{2} + ip - 2n\frac{1}{2Q} = -\frac{Q}{2}, \quad \text{or} \quad iQp = n .$$ (2.17)

Thus, the residues of the poles at $iQp = n$ can be calculated by evaluating Veneziano amplitudes with $2n$ insertions of the boundary superpotential (2.14) and its complex conjugate. In this calculation it is important to include the boundary fermionic degrees of freedom that we omitted in (2.14).

The other set of poles of (2.6) occurs when $\frac{2ip}{Q}$ is a positive integer. The residues of these poles can be computed by expanding the boundary action in the perturbation (2.4) to n'th order, and imposing the anomalous conservation condition

$$-\frac{Q}{2} + ip - n\frac{Q}{2} = -\frac{Q}{2}, \quad \text{or} \quad \frac{2ip}{Q} = n .$$ (2.18)

We see that in order to reproduce the analytic structure of the full amplitude (2.6) we need to take into account both the tachyon condensate and the curved shape of the brane.

One can also extend the analysis of [35] to this case and derive the one point function (2.6) by using degenerate operators. As there, this should give a relation between λ in (2.4) and $\tilde{\lambda}$ in (2.14). We will not perform this calculation (or that of the residues of the poles discussed in the previous two paragraphs) here. It would be nice to check that it works.

Our main interest is in understanding the nature of α' corrections to the classsical hairpin shape (2.3). To do this, we need the exact real space wave function of the hairpin, which is obtained by Fourier transforming the disk one point function (2.6). This computation was done in [5]. The result can be written in terms of the variable

$$y = \frac{e^{-\frac{Q}{2}\phi}}{2\cos\frac{Q}{2}x}$$ (2.19)

as follows:

$$\Psi(\phi, x) \simeq \frac{y^{\frac{2}{Q^2}} e^{-y^{\frac{2}{Q^2}}}}{2\cos\frac{Q}{2}x} .$$ (2.20)

For later use in Minkowski spacetime, it is useful to compute the (xx) component of the stress tensor associated with the brane, which is [7]:

$$T_{xx} \simeq -y^{\frac{2}{Q^2}-1} e^{-y^{\frac{2}{Q^2}}} \ . \tag{2.21}$$

We see that the effect of α' corrections is to replace the classical solution (2.3), which has a particular value of y, $y = 1$, with a smeared solution, in which y has a finite spread.

The properties of the smeared solution depend on Q. For small Q, the stress tensor (2.21) is sharply peaked at

$$y = \frac{e^{-\frac{Q}{2}\phi}}{2\cos\frac{Q}{2}x} = \left(1 - \frac{Q^2}{2}\right)^{\frac{Q^2}{2}} = 1 - \frac{Q^4}{4} + \cdots \ . \tag{2.22}$$

As the linear dilaton slope Q increases, the maximum of the distribution moves to smaller y, or larger ϕ. The distribution also becomes broader. When $Q^2 \to 2$, the peak of the distribution goes to infinite ϕ, and its width diverges.

The spread of ϕ can be read off (2.19):

$$\delta\phi \simeq \frac{2}{Q}\delta\ln y \ . \tag{2.23}$$

For small Q, $y \simeq 1$, (2.22), while the width of the distribution (2.20) is $\delta y \simeq Q^2$. Hence the spread of ϕ is

$$\delta\phi \sim \frac{1}{Q}\frac{\delta y}{y} \sim Q \ . \tag{2.24}$$

As $Q^2 \to 2$, one finds that $\delta\phi$ diverges.

The above properties of the boundary state can be understood in the following way. We saw that the main correction to the DBI picture of a brane with the shape (2.3) is the condensate of the tachyon stretched between the two sides of the hairpin. For small Q this tachyon is asymptotically very massive due to the large distance over which it is stretched (2.13). Therefore, its condensate (2.14) goes rapidly to zero as $\phi \to \infty$, and the asymptotic part of the hairpin is not significantly modified by its presence. As we get closer to the tip of the hairpin, the tachyon becomes lighter, its condensate grows and its effects increase. It makes the location of the tip fuzzy, and pulls it to larger ϕ (2.22).

An instructive way to estimate the size of the region in which the stretched tachyon condensate is significant, is the following. The quadratic term in the effective action of the tachyon $T(\phi)$ has the form

$$\mathcal{L} = -e^{\frac{Q}{2}\phi}\left[(\partial_\phi T)^2 + m^2 T^2\right] \ . \tag{2.25}$$

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The tachyon mass is

$$m^2 = -\frac{1}{4} + \left(\frac{x}{2\pi}\right)^2 \; , \tag{2.26}$$

where the first term is the standard mass of the open string tachyon in fermionic string theory, and the second is due to the stretching of the tachyon over a distance $2x$ (at a given ϕ). This distance depends on ϕ via the hairpin shape (2.3). As ϕ varies between infinity and its minimal value, x changes between $\frac{\pi}{Q}$ and zero.

It is convenient to perform the field redefinition

$$T(\phi) = e^{-\frac{Q}{4}\phi}\widetilde{T}(\phi) \; , \tag{2.27}$$

to make the kinetic term in (2.25) canonical. In terms of $\widetilde{T}$, the effective Lagrangian has the form

$$\mathcal{L} = -(\partial_\phi \widetilde{T})^2 - \widetilde{m}^2(\phi)\widetilde{T}^2 \; , \tag{2.28}$$

where the effective mass $\widetilde{m}$ is given by

$$\widetilde{m}^2(\phi) = m^2(\phi) + \frac{Q^2}{16} \; . \tag{2.29}$$

As $\phi \to \infty$, the effective mass approaches the value

$$\widetilde{m}_\infty = \frac{1}{2}\left(\frac{1}{Q} - \frac{Q}{2}\right) \; . \tag{2.30}$$

As we approach the tip the contribution from stretching goes to zero, and the effective mass goes to

$$\widetilde{m}_0^2 = -\frac{1}{4} + \frac{Q^2}{16} \; . \tag{2.31}$$

This value is negative for all $Q^2 < 4$ (*i.e.* in all situations of interest except two dimensional string theory), reflecting the fact that the open string tachyon is tachyonic above two dimensions.

The equation of motion of the tachyon $\widetilde{T}$ is

$$-\partial_\phi^2 \widetilde{T} + \widetilde{m}^2(\phi)\widetilde{T} = 0 \; . \tag{2.32}$$

This looks like a Schroedinger equation for a particle living in the potential $V(\phi) = \widetilde{m}^2(\phi)$, with energy $E = 0$. The analog of the Schroedinger wave function is $\widetilde{T}$, which is indeed the wave function of the tachyon. Such a wave function spreads over the region in ϕ in which the potential $V(\phi)$ is smaller than the energy, or in other words over the region in

which the effective mass squared of the tachyon (2.29) is negative. For small Q the size of this region goes like $\delta\phi \sim Q$, while for $Q^2 \to 2$ it diverges like

$$\delta\phi \sim -2\sqrt{2}\ln\widetilde{m}_\infty \ , \tag{2.33}$$

in agreement with our analysis of the boundary state, in the discussion following eq. (2.23).

We are thus led to the following picture. When the linear dilaton slope Q is very small, the effective mass at infinity, (2.30), is very large, and the tachyon wave function $\widetilde{T}$ decays like

$$\widetilde{T} \sim e^{-\widetilde{m}_\infty \phi} \tag{2.34}$$

for large ϕ. As we go to smaller ϕ, the mass squared of the tachyon decreases and eventually becomes negative, in a region of size $\delta\phi \sim Q$ around the turning point. In that region the condensate of the tachyon is not suppressed. Its effect is to smear the hairpin, and shift its tip to slightly larger ϕ.

As Q increases, the size of the region where the tachyon is light grows, and the smearing fo the hairpin brane due to the light tachyon increases. The amount of smearing for Q^2 slightly below two is given by (2.33). As $Q^2 \to 2$, $\widetilde{m}_\infty$ (2.30) goes to zero, and the fluctuations extend all the way to infinity (2.33). At that point, one can no longer think about the boundary state in terms of the original hairpin shape.

Earlier in this section (after eq. (2.3)) we mentioned that, just like for the cigar geometry, the shape of the hairpin brane is not expected to receive α' corrections in the fermionic string. We now see in what sense this is correct. The boundary state certainly *does* receive α' corrections, which are, for example, visible in the form of the one point function (2.6). As we discussed, these corrections are due to the stretched tachyon condensate (2.14). However, in position space the corrections are relatively mild: they do not change the asymptotic separation between the two $D1$-branes – (2.13) is exact. Also, the hairpin equation (2.3) is still valid in an average sense; *e.g.* we saw that the expectation value of the stress tensor is peaked at (2.22), which again has the hairpin shape (2.3), with a particular value of the constant C.

3. Rolling D-brane

In this section we discuss the Minkowski continuation of the hairpin brane, which is obtained by replacing $x \to it$ in (2.3). This leads to the trajectory

$$e^{-\frac{Q}{2}\phi} = 2C\cosh\frac{Q}{2}t \ , \tag{3.1}$$

corresponding to a D-brane localized in ϕ, which starts at early times deep inside the throat with a speed close to that of light, rises to some maximal height and then falls back towards the strong coupling region $\phi \to -\infty$. Alternatively, one can think of (3.1) for $t \geq 0$ as describing a process where the D-brane is released from rest at $t = 0$, and rolls down the throat. The parameter C can again be set to one by shifting ϕ.

Unlike the Euclidean hairpin (2.3), which never makes it into the strong coupling region $\phi \to -\infty$, the rolling D-brane trajectory (3.1) visits this region at very early and very late times. One might worry that the boundary CFT describing the rolling brane will be singular, like the linear dilaton theory itself. A quick way to see that this is not the case [3] is that the worldsheet CFT is typically singular when D-branes are light. As long as the energy of the rolling D-brane in string units is large, whether that energy is potential or kinetic, we do not expect the theory on the disk to exhibit any pathologies. Indeed, the boundary state obtained by Wick rotation of the hairpin brane appears to be well behaved.

There are potential divergences in loop amplitudes [5,6], which are associated with processes in which the D-brane loses its energy to closed string radiation and becomes light at late times. We will focus on phenomena that occur at shorter time scales, and will not discuss these quantum effects here.

As for the hairpin, we expect the trajectory (3.1) to receive stringy corrections for finite Q, and our purpose is to study their nature. In Euclidean space, we saw that corrections to the hairpin shape (2.3) were due to the condensate of the tachyon (2.14). A direct Wick rotation of this condensate seems subtle since the tachyon depends on the T-dual of x, which does not have a simple continuation to Minkowski time. Therefore, our strategy will be to examine the Minkowski boundary state, and use it to interpret the string corrections.

Given the discussion of the Euclidean case, this is rather straightforward to do. As is familiar from other contexts such as the rolling tachyon boundary state (see [43] for a

review), a natural way to perform the Wick rotation is in position space [5]. Thus, we define a Minkowski version of the variable y (2.19),

$$y_m = \frac{e^{-\frac{Q}{2}\phi}}{2\cosh\frac{Q}{2}t} \tag{3.2}$$

and use (2.21) to compute the energy density of the rolling D-brane, [7],

$$T_{00} = \frac{E}{Q\Gamma(1 - \frac{Q^2}{2})} y_m^{\frac{2}{Q^2} - 1} e^{-y_m^{\frac{2}{Q^2}}} , \tag{3.3}$$

where E is the total energy of the brane.

In a similar way to the Euclidean analysis of section 2, one can use (3.3) to find the location of the D-brane as a function of time. In the small slope limit $Q \to 0$ one finds that the distribution of energy density (3.3) is sharply peaked at $y_m = 1$, the classical trajectory (3.1). As Q increases, the distribution of y_m becomes wider, and its peak moves to smaller values (2.22).

The smearing of the trajectory (3.1) is related to that of y_m as in (2.23),

$$\delta\phi \simeq \frac{2}{Q}\delta \ln y_m . \tag{3.4}$$

In particular, the spread of ϕ is independent of time, as in the Euclidean case, where it was independent of x. The smearing increases with the dilaton slope, and pushes the top of the trajectory, $\phi(t = 0)$, out into the weak coupling region. As in the Euclidean case, for small Q one finds that the smearing is small, $\delta\phi \sim Q$, while as $Q^2 \to 2$, the top of the trajectory is pushed to infinite ϕ, and the D-brane is completely delocalized. For larger dilaton slope, the Minkowski boundary state does not seem to describe the motion of a localized object in the throat. For example, the integral giving the total energy,

$$E = \int_{-\infty}^{\infty} d\phi T_{00}(\phi, t) , \tag{3.5}$$

diverges for $Q^2 \geq 2$. The divergence comes from the region $\phi \to \infty$.

The hairpin, (2.3), which is the Euclidean continuation of the Minkowski solution (3.1) has a periodicity in the Euclidean time direction, $x \sim x + \frac{4\pi}{Q}$. This means that the natural Minkowski continuation corresponds to a D-brane in a heat bath at temperature

$$T_u = \frac{Q}{4\pi} . \tag{3.6}$$

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This temperature has a natural interpretation in terms of the dynamics of the brane. To see that, we recall the Unruh effect [44], according to which an accelerated observer experiences a thermal bath of particles at temperature

$$T_u = \frac{a}{2\pi} \ , \tag{3.7}$$

where a is the acceleration of the observer. The rolling D-brane (3.1) can be thought of as an Unruh observer. As it starts from rest at $t = 0$, its acceleration is given by

$$a_0 = -\frac{Q}{2} \ . \tag{3.8}$$

The corresponding Unruh temperature (3.7) is given by (3.6). Thus, we see that the heat bath in which the D-brane is immersed has a temperature equal to its Unruh temperature at the top of the trajectory.

In the Euclidean case, the string corrections to the hairpin geometry were due to the stretched tachyon (2.14). An interesting question, to which we turn next, is what is the interpretation of the corresponding corrections to the rolling D-brane trajectory (3.1).

One way of thinking about this problem is to consider the process in which the D-brane is released from rest at $t = 0$, and, without taking into account string corrections, accelerates according to (3.1). The Wick rotation from Euclidean space can be thought of as specifying an initial state for the rolling D-brane, which can be obtained by gluing the Euclidean hairpin solution to the Minkowski solution at $t = 0$. This is an analog of the Hartle-Hawking prescription for black holes.

The fact that the hairpin brane has a non-zero open string tachyon condensate, which is localized in the vicinity of the tip, suggests that the Minkowski initial state at $t = 0$ contains some open strings. The form of the energy density (3.3) suggests that these open strings oscillate in the ϕ direction, giving the stringy halo around the classical trajectory. Due to the acceleration of the D-brane, the open strings experience a force in the direction opposite the acceleration, $i.e.$ in the positive ϕ direction. Therefore, we expect the stringy smearing due to these excited strings to be skewed towards larger values of ϕ.

This is precisely what we see in the distribution of energy density (3.3). The suppression at large y_m (which corresponds to $\phi \to -\infty$) is exponential, while that at small y_m (or $\phi \to \infty$) is only powerlike, with a power that decreases as the acceleration (3.8) grows. Therefore, the tail of the distribution at large positive ϕ is much larger than that at large negative ϕ.

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We are led to the following picture. For small Q, the excited open strings give rise to a small spread in ϕ, (3.4), by an amount $\delta\phi \sim Q$, proportional to the temperature (3.6), and the top of the trajectory is pushed to slightly larger ϕ by the amount (2.22).

As Q increases, the acceleration (3.8) grows, and eventually the Unruh temperature (3.6) reaches the Hagedorn temperature of perturbative string theory in the linear dilaton throat (2.1), (2.2). The latter is given by (see *e.g.* [45])

$$T_h = \frac{1}{2\pi\sqrt{4-Q^2}} \; . \tag{3.9}$$

Thus, $T_u = T_h$ when $Q^2 = 2$. At that point, one expects the effect of the classical condensate of open strings to be large. Indeed, the exact boundary state has the property that as $Q^2 \to 2$, the trajectory develops large smearing, is pushed out to infinite ϕ, and becomes non-normalizable.

One can write the asymptotic mass of the winding tachyon (2.30), in the following suggestive way:

$$\widetilde{m}_\infty^2 = \left(\frac{1}{8\pi T_u}\right)^2 - \left(\frac{1}{8\pi T_h}\right)^2 = \left(\frac{\beta_u}{8\pi}\right)^2 - \left(\frac{\beta_h}{8\pi}\right)^2 \; . \tag{3.10}$$

This formula is very reminiscent of the one for the mass of the thermal scalar in ordinary flat space string theory at finite temperature. There, it is usually said that the thermal scalar encodes the properties of highly excited perturbative strings, which dominate the critical behavior near the Hagedorn temperature. Similarly, in our case the condensate of the stretched tachyon (2.14) in Euclidean space encodes the effects of the condensate of excited open strings, which for $T_u \to T_h$ consists mainly of very long strings. These strings cause the large smearing of the trajectory of the D-brane, which in the Euclidean case is attributed to the condensate of the stretched tachyon. One can describe the situation by saying that the Wick rotation of the stretched tachyon condensate to Minkowski space *is* the excited open strings.

4. Black holes

In this section we will generalize the considerations of the previous sections to the case of black hole solutions in string theory. We start with a discussion of the two dimensional Euclidean and Minkowski $SL(2, \mathbb{R})/U(1)$ black hole, where α' effects are understood and we are on firmer ground. In subsection 4.2 we propose a generalization to other black holes, where exact solutions are in general not available and our discussion will be more tentative. We will restrict to the case of uncharged black holes. It would be interesting to generalize the discussion to charged ones.

4.1. Two dimensional black hole

Euclidean $SL(2, \mathbb{R})/U(1)$ CFT is a solution of the classical equations of motion of string theory in asymptotically linear dilaton space [46,47,48]. The metric takes the cigar form

$$ds^2 = d\phi^2 + \left(\tanh^2 \frac{Q}{2}\phi \right) d\theta^2 \ , \tag{4.1}$$

where θ is a periodic coordinate,

$$\theta \sim \theta + \frac{4\pi}{Q} \ , \tag{4.2}$$

and $0 \leq \phi < \infty$. The dilaton is

$$D = D_0 - \ln \cosh \frac{Q}{2}\phi \ . \tag{4.3}$$

D_0 is the value of the dilaton at the tip of the cigar. It is a free parameter, which should be chosen large and negative, such that the string coupling at the tip, $\exp D_0$, is small.

As mentioned in the introduction, in addition to the metric and dilaton (4.1), (4.3), this CFT has a condensate of a closed string tachyon wrapped around the cigar [31,32,33], which corresponds to the $N = 2$ Liouville superpotential

$$W = \mu e^{-\frac{1}{Q}\Phi} \ , \tag{4.4}$$

with $\Phi = \phi + i\tilde{\theta}$, and $\tilde{\theta}$ the T-dual of θ. As in the boundary case (2.15), for small Q the tachyon background goes rapidly to zero at large ϕ, and does not play much of a role there. However, as Q increases, it becomes more important, until it takes over at $Q^2 = 2$ and turns the cigar CFT into a non-normalizable deformation of the linear dilaton one.

One way to see the effects associated with the superpotential (4.4) is to study the two point function of the tachyon vertex operators (2.5) in the cigar background. It is given by [31,33,49]

$$\langle T(p,q)T(-p,-q)\rangle \simeq \frac{\Gamma(-\frac{2ip}{Q})}{\Gamma(\frac{2ip}{Q})} \frac{\Gamma^2(\frac{1}{2} + \frac{ip}{Q} + \frac{q}{Q})}{\Gamma^2(\frac{1}{2} - \frac{ip}{Q} + \frac{q}{Q})} \frac{\Gamma(1 - iQp)}{\Gamma(1 + iQp)} \ . \tag{4.5}$$

One can think of (4.5) as the $1 \rightarrow 1$ scattering matrix from the tip of the cigar and/or $N = 2$ Liouville potential. As discussed in [35,37], some of the poles of (4.5) can be understood by expanding the two point function in a power series in (4.4), and in the metric deformation associated with (4.1) at large ϕ.

The two point function (4.5) has a qualitatively similar structure to that of the one point function (2.6). The first two ratios of Γ-functions depend on $\frac{p}{Q}$ and $\frac{q}{Q}$. This is an analog of the classical wave function (2.9). In the present case, one can show [40] that this is what one gets from classical geometry, by studying $1 \to 1$ scattering of fields in the geometry (4.1), (4.3).

The last ratio of Γ-functions in (4.5) depends on Qp. Its effects smear the cigar geometry in the ϕ direction. To determine the size of this smearing, one can proceed as in section 2. The quadratic term in the effective Lagrangian for the winding tachyon T has the form

$$\mathcal{L} = \cosh^2 \left(\frac{Q}{2}\phi \right) \left[(\partial_\phi T)^2 + m^2(\phi)T^2 \right] \ , \tag{4.6}$$

where the ϕ-dependent mass of T is

$$m^2(\phi) = -1 + \frac{1}{Q^2} \tanh^2 \frac{Q}{2}\phi \ . \tag{4.7}$$

The first term in (4.7) is the standard mass of the closed string tachyon in the fermionic string, while the second is the contribution due to winding around the θ circle.

The wave function of the tachyon, $\widetilde{T}$, is obtained by dividing the tachyon field T by the local string coupling, $g_s(\phi)$, as in (2.27),

$$\widetilde{T}(\phi) = T\cosh\frac{Q}{2}\phi \ . \tag{4.8}$$

In terms of $\widetilde{T}$, the Lagrangian (4.6) takes the form

$$\mathcal{L} = (\partial_\phi \widetilde{T})^2 + \widetilde{m}^2(\phi)\widetilde{T}^2 \ , \tag{4.9}$$

where the effective mass $\widetilde{m}$ is

$$\widetilde{m}^2 = \frac{Q^2}{4} - 1 + \frac{1}{Q^2} \tanh^2 \frac{Q}{2}\phi \ . \tag{4.10}$$

As $\phi \to \infty$, it goes to

$$\widetilde{m}_\infty = \frac{1}{Q} - \frac{Q}{2} \ , \tag{4.11}$$

while for $\phi \to 0$ one finds

$$\widetilde{m}_0^2 = -1 + \frac{Q^2}{4} \ . \tag{4.12}$$

Note that both $\widetilde{m}_\infty$ and $\widetilde{m}_0$ are twice as large as the corresponding open string quantities (2.30) and (2.31). In particular, $\widetilde{m}_0^2$ is negative.

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The equation of motion of $\widetilde{T}$ is again given by (2.32), and looks like the Schroedinger equation for a zero energy wave function in the potential $\widetilde{m}^2(\phi)$. The wave function spreads over the region $0 \le \phi \le \phi_0$, in which $\widetilde{m}^2(\phi)$ is negative. From (4.10) we see that ϕ_0 is given by

$$\frac{1}{Q^2} \tanh^2 \frac{Q}{2} \phi_0 = 1 - \frac{Q^2}{4} \ . \tag{4.13}$$

For small Q, one has $\phi_0 = 2 + O(Q^2)$. Note that this is different from the open string case, where the spread of the hairpin for small Q was by an amount proportional to Q, (2.24). As $Q^2 \to 2$, ϕ_0 diverges in a way similar to (2.33),

$$\phi_0 \sim -\sqrt{2} \ln \widetilde{m}_\infty \ . \tag{4.14}$$

Thus, we are led to the following picture. For a fixed value of $Q < \sqrt{2}$, far from the tip of the cigar, the $SL(2, \mathbb{R})/U(1)$ CFT is well described by the metric and dilaton (4.1), (4.3). However, in a region of size ϕ_0, (4.13), there are large fluctuations of the geometry, which are due to the presence of a light winding tachyon condensate (4.4). The size of this region grows with Q, and diverges as $Q^2 \to 2$, (4.14), where the cigar CFT ceases to be normalizable. We see that the picture is very similar to the one obtained for the hairpin brane in section 2.

The Minkowski two dimensional black hole is obtained by Wick rotating $\theta \to it$ in (4.1). It can also be studied algebraically by using its $SL(2, \mathbb{R})/U(1)$ description. The Hawking temperature of this black hole is

$$T_{bh} = \frac{Q}{4\pi} \ . \tag{4.15}$$

Note that (4.15) is equal to the Unruh temperature of the rolling D-brane discussed in section 3, (3.6). T_{bh} also has the interesting property that it is independent of the mass of the black hole. This too has an analog in the rolling D-brane problem, for which the Unruh temperature (3.6) is independent of the energy of the D-brane.

Due to the close analogy to the rolling D-brane, we can borrow most of the discussion of section 3 to the present case. The condensate of the winding tachyon in the Euclidean black hole solution (4.1) corresponds in Minkowski space to a condensate of closed strings near the horizon of the black hole. This leads to the smearing of the region near the horizon, and the appearance of a stretched horizon of the sort discussed in [38]. The size of this stretched horizon is given by (4.13).

This size has a very natural Minkowski interpretation. While the Hawking temperature seen by an observer at infinity is given by (4.15), an observer stationary at a fixed value of ϕ sees a higher temperature [50], due to the red-shift factor in (4.1). That temperature is given by

$$T_{bh}(\phi) = \frac{Q}{4\pi} \frac{1}{\tanh \frac{Q}{2}\phi} \ . \tag{4.16}$$

Even if the Hawking temperature at infinity (4.15) is low, the local temperature (4.16) increases as one approaches the horizon, where it formally diverges. It is natural to expect that stringy effects will smear the part of the geometry for which the local Hawking temperature is higher than the Hagedorn one [38]. Comparing (4.16) to (3.9) we see that the place where the two coincide is precisely (4.13). As we saw before, the size of the stretched horizon is of order one in string units for black holes with very low Hawking temperature, and it diverges as the Hawking temperature at infinity approaches the Hagedorn temperature.

In the rolling D-brane case we described the situation by saying that the Minkowski continuation of the stretched tachyon condensate (2.15) is the classical condensate of open strings that smears the trajectory of the D-brane. Similarly, we can say that the continuation of the winding tachyon (4.4) is the classical condensate of closed strings near the horizon, which smears the geometry in a region of size (4.13). The mass of the wound tachyon (4.11) can be written in a way analogous to (3.10),

$$\widetilde{m}_\infty^2 = \left(\frac{1}{4\pi T_{bh}}\right)^2 - \left(\frac{1}{4\pi T_h}\right)^2 = \left(\frac{\beta_{bh}}{4\pi}\right)^2 - \left(\frac{\beta_h}{4\pi}\right)^2 \ , \tag{4.17}$$

which is also suggestive of this interpretation, as in the discussion after (3.10).

The notion that a small black hole has a stretched horizon whose size is much larger than the horizon size sounds counter-intuitive. The following remarks may help clarify this issue.

In a black hole spacetime, one can divide the modes of quantum fields into two classes. One consists of modes that originate far from the black hole; these are refered to in [50] as "past-infinity" modes. The other consists of modes that appear to an outside observer to emanate from the black hole. These are the so called "white hole" modes. For a black hole in vacuum, the white hole modes are in a thermal state at their Hawking temperature (4.15), (4.16), but the past infinity modes are in their vacuum state. On the other hand, for a black hole in thermal equilibirium with a heat bath, both kinds of modes are in a thermal state at the relevant temperature.

An observer at the distance (4.13) from a two dimensional black hole will perceive thermal radiation at the Hagedorn temperature coming out of the black hole, but depending on the presence or absence of a heat bath at the Hawking temperature, will or will not feel thermal radiation travelling towards the hole. In $1+1$ dimensions, the difference between the two situations seems to be small. Thus, in this case, an observer at a fixed ϕ will indeed perceive the local Hawking temperature (4.16), even if the radiation from infinity is absent.

In higher dimensions, the case discussed in the next subsection, the difference is bigger (for small black holes), since the radiation in white hole modes disperses over the angular spheres. If the thermal bath is absent, an observer at a given radial distance may see very faint radiation at close to the Hagedorn temperature, but will not feel that the local temperature around him is T_h. The continuation from Euclidean space naturally describes the black hole in thermal equilibrium with a heat bath, and in that system, one can say that the effective size of the black hole diverges when the Hawking temperature approaches the Hagedorn one.

4.2. Higher dimensional black holes

In the previous subsection we identified the mechanism for the appearance of a stretched horizon for classical two dimensional black holes. According to [38], a similar stretched horizon is expected to surround general black holes in string theory. It is natural to ask whether our arguments can be generalized to other types of black holes, such as Schwarzschild. In this subsection we discuss this question.

Consider, as an example, a Schwarzschild black hole in asymptotically flat four dimensional spacetime.[3] We will assume that the classical equations of motion of string theory have a solution which looks like an α' corrected Schwarzschild solution. In other words, we are assuming the existence of a one parameter set of solutions labeled by the mass, which for large masses approaches the Schwarzschild solution, but also makes sense for masses for which the Hawking temperature is of order the Hagedorn temperature. The basic properties we require of such a solution are spherical symmetry, the existence of a horizon, and of a Wick rotated Euclidean solution, in which Euclidean time is compactified on a circle with asymptotic circumference $\beta_{bh} = 1/T_{bh}$. The circle should be contractible, *i.e.* the winding number around it should not be conserved.

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[3] With the six extra dimensions compactified.

It is possible that this assumption is incorrect, but we do not know of any reason why this should be the case, and the two dimensional black hole provides an existence proof that such solutions are possible. It is very natural for such solutions to exist, since the masses for which the Hawking temperature is of order T_h go like $1/g_s^2$, and spherically symmetric states with such masses should be described in the limit $g_s \to 0$ by non-trivial closed string geometries. Moreover, such solutions should have horizons since they should give rise to the large entropy of states with these masses. In any case, we will assume that they exist and proceed.

The Hawking temperature of the black hole, T_{bh}, decreases as its mass increases, and for large enough mass can be made arbitrarily small compared to the Hagedorn temperature. However, this is the temperature at infinity. An observer at a fixed proper distance R from the horizon sees a higher Hawking temperature,

$$T_{bh}(R) = \frac{T_{bh}}{\chi(R)} \ , \tag{4.18}$$

where χ is the redshift factor in the black hole metric, $\chi = \sqrt{g_{00}}$. For large R, $\chi \to 1$, while as R decreases, χ does as well, so the local Hawking temperature (4.18) increases. In particular, as $R \to 0$, the local Hawking temperature formally diverges. For small R, $T_{bh}(R)$ is more naturally thought of as an Unruh temperature (3.7), due to the acceleration of an observer held at a fixed distance from the horizon; see $e.g.$ [50] for a discussion.

When the local Hawking temperature (4.18) approaches the Hagedorn temperature in flat spacetime,[4]

$$T_h = \frac{1}{4\pi} \ , \tag{4.19}$$

one expects stringy effects to become important. These effects are expected to smear the geometry into a stretched horizon [38], which can be defined as the region where

$$T_{bh}(R) \geq T_h \ . \tag{4.20}$$

For a large black hole, the size of the stretched horizon is of order one in string units (see below).

As the mass of the black hole decreases, and the corresponding Hawking temperature increases, we encounter a conundrum. On the one hand, as the mass decreases, the gravity fields decay more rapidly at infinity. In particular, the Einstein gravity approximation to

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[4] Which is given by the limit $Q \to 0$ of (3.9).

the full solution becomes better and better for any fixed radial distance from the black hole, and the geometry should look more and more like flat space there. On the other hand, as the Hawking temperature increases, we expect the stretched horizon defined via (4.20) to increase in size, since a smaller redshift factor is needed to bridge the gap between the temperature at infinity, T_{bh}, and the Hagedorn temperature, T_h. If the gravity solution becomes more and more like flat space, what causes the smearing of the geometry at long distances that is implied by a large stretched horizon?

The discussion of the previous subsection suggests an obvious answer. The Euclidean Schwarzschild geometry contains not just the gravity fields that are usually considered, but also a condensate of a tachyon wrapped around the Euclidean time circle. For very massive black holes, this tachyon is asymptotically very heavy, due to the large radius of Euclidean time, and so its effects at infinity are very small. However, as in our discussion of the hairpin brane in section 2, and of the cigar in the previous subsection, there is a small region near the horizon in which this tachyon is light, and its effects lead to a smearing of the geometry there.

As the Hawking temperature increases, the asymptotic mass of the tachyon decreases, and the size of the stretched horizon grows, until for $T_{bh} \to T_h$ it diverges, and the black hole ceases to be normalizable.

To illustrate the above ideas, we will generalize the calculations of the previous subsection to the four dimensional Schwarzschild black hole case. In order to study the geometry at the string scale, we need the exact solution of classical string theory corresponding to this black hole, which is at present unavailable. In the spirit of [39], we will use the leading order solution (*i.e.* the original Schwarzschild one), and hope that the α' corrections do not modify the discussion too much. Of course, for any black hole for which the exact solution is available, we can do better.

The line element of the four dimensional Euclidean Schwarzschild solution is

$$ds^2 = \left(1 - \frac{r_0}{r}\right) d\theta^2 + \frac{1}{1 - \frac{r_0}{r}} dr^2 + r^2 d\Omega \ , \tag{4.21}$$

where r_0 is the Schwarzschild radius, and θ is the Euclidean time coordinate. It is periodically identified $\theta \sim \theta + \beta_{bh}$, where β_{bh} is the inverse Hawking temperature. The different parameters are determined by the mass of the black hole as follows:

$$\begin{aligned} r_0 &= 2MG_N \ , \\ \beta_{bh} &= 4\pi r_0 = 8\pi MG_N \ , \end{aligned} \tag{4.22}$$

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where G_N is Newton's constant. The inverse Hagedorn temperature of string theory in flat spacetime is given by $\beta_h = 4\pi$, (4.19). Hence, if we trust the leading order solution (4.21), the Hawking temperature (4.22) is lower than the Hagedorn one for $r_0 > 1$.

The boundary conditions around the θ circle at $r \to \infty$ are such that a tachyon wrapped once around the circle survives the GSO projection. As discussed above, it is natural to expect that this tachyon has a non-zero expectation value in the solution corresponding to (4.21) in string theory. To understand where in r that expectation value is supported, we examine the quadratic terms in the action for the tachyon. Taking the tachyon to be an s-wave on the angular two sphere, the action takes the form (up to an overall constant, which will not play a role)

$$S = \int_{r_0}^{\infty} dr\, r^2 \left[\left(1 - \frac{r_0}{r} \right) (\partial_r T)^2 + m^2(r) T^2 \right] . \tag{4.23}$$

The mass $m(r)$ has again two contributions, the usual closed string tachyon mass and a term from winding around the circle:

$$m^2(r) = -1 + r_0^2 \left(1 - \frac{r_0}{r} \right) . \tag{4.24}$$

As $r \to \infty$, the mass approaches

$$m_\infty^2 = r_0^2 - 1 . \tag{4.25}$$

Since we are assuming that the asymptotic Hawking temperature (4.22) is below the Hagedorn temperature (4.19), m_∞^2 is positive. As $r \to \infty$, T behaves as a standard scalar field with mass (4.25), and its condensate decays like

$$T(r) \sim \frac{1}{r} e^{-m_\infty r} . \tag{4.26}$$

On the other hand, as $r \to r_0$, the mass (4.24) goes to $m^2 \to -1$. The tachyon condensate is a solution of the equation of motion of (4.23), and we expect it to have some finite spread in the r direction. To find this spread it is convenient to first change coordinates to

$$e^z = 1 - \frac{r_0}{r} . \tag{4.27}$$

In the new coordinates, the horizon corresponds to $z = -\infty$, while $r = \infty$ is mapped to $z = 0$.

In these coordinates, the action (4.23) takes the form

$$S = r_0 \int_{-\infty}^{0} dz \left[(\partial_z T)^2 + \widetilde{m}^2(z) T^2 \right] , \tag{4.28}$$

where

$$\widetilde{m}^2(z) = r_0^2 \frac{e^z}{(1 - e^z)^4} (r_0^2 e^z - 1) . \tag{4.29}$$

As in the other cases described above, the tachyon wave function spreads over the region in which $\widetilde{m}^2$ is negative,

$$r_0^2 e^z < 1 . \tag{4.30}$$

In terms of the original r coordinates (4.27), this region is

$$1 - \frac{r_0}{r} < \frac{1}{r_0^2} . \tag{4.31}$$

By using (4.18), (4.19), (4.22), one can check that (4.31) is precisely the region (4.20), in which the local Hawking temperature is higher than the Hagedorn temperature.

If the black hole is very massive, such that $r_0 \gg 1$, it is more natural to express (4.31) in terms of the proper distance from the horizon, R. For $r - r_0 \ll r_0$, the proper distance is given by

$$R = 2r_0 \sqrt{1 - \frac{r_0}{r}} . \tag{4.32}$$

Hence, we can write (4.31) as $R < 2$. We see that for large black holes the stretched horizon has size of order one in string units. In the other extreme limit, when the Hawking temperature of the black hole approaches the Hagedorn temperature, $i.e.$ $r_0 \to 1$, the size of the stretched horizon diverges. Indeed, in that limit (4.31) takes the form

$$r < \frac{1}{2(r_0 - 1)} . \tag{4.33}$$

Note that while the detailed estimate of the size of the stretched horizon for very massive black holes relies on the structure of the (unknown) α' corrections to the solution (4.21), the estimate (4.33) does not. Indeed, in the limit $r_0 \to 1$ the tachyon mass (4.25) goes to zero, so the effective action (4.23) becomes more and more accurate. Furthermore, at the distance scale (4.33), the metric is essentially flat, so α' corrections to the solution should not play a role.

To find the energy at which the size of the stretched horizon diverges, we need to know the relation between the mass and r_0 which generalizes (4.22). This does require

knowledge of the α' correction. If we knew that relation, we would be able to calculate the precise energy at which the black hole becomes non-normalizable. This should also be the point at which the string/black hole transition occurs, since when the Hawking and Hagedorn temperatures approach each other, it becomes more and more difficult to distinguish the black hole with its infinitely large stretched horizon from a string state with the same energy. Therefore, at that point, the Bekenstein-Hawking and string entropies must agree.

To get a feeling for how this works, imagine that the Schwarzschild geometry (4.21) received no α' corrections (an assumption that is surely incorrect). Then we would proceed as follows. We saw that the Hawking and Hagedorn temperatures coincide when the horizon size $r_0 = 1$. At that point, the Bekenstein-Hawking entropy of the black hole is given by

$$S_{bh} = \frac{A}{4G_N} = \frac{\pi r_0^2}{G_N} = \frac{\pi}{G_N} \ . \tag{4.34}$$

The mass of the black hole is given by (4.22)

$$M = \frac{1}{2G_N} \ . \tag{4.35}$$

Substituting this into (4.34) we see that the Bekenstein-Hawking entropy at the transition point is given by

$$S_{bh} = 2\pi M \ . \tag{4.36}$$

On the other hand, the fundamental string entropy is given by (4.19)

$$S_f = 4\pi M \ . \tag{4.37}$$

We see that the two differ by a factor of two,[5] which is very likely due to higher α' corrections. One can try to study this systematically by including (tree level) higher curvature corrections to the Einstein equations, but we will not attempt to do this here.

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[5] For a d dimensional Schwarzschild black hole, this factor is $(d-2)/(d-3)$.

5. Discussion

In this paper we studied the α' effects in some Euclidean and Lorentzian backgrounds of classical string theory. One of the solutions we discussed corresponds to a D-brane rolling in a linear dilaton space. We argued that the main string correction to the trajectory of such a D-brane is a stringy smearing due to the presence of oscillating open strings attached to the brane. These effects increase with the gradient of the dilaton, and diverge at a critical value of the gradient, at which the Unruh temperature of the brane reaches the Hagedorn temperature of perturbative string theory in the linear dilaton background. At that point the trajectory of the D-brane is completely smeared, and the brane is pushed out of the linear dilaton throat.

The Euclidean solution corresponding to the rolling D-brane (the hairpin brane) encodes these effects via the condensate of an open string tachyon stretched in the Euclidean time direction. In Euclidean space this tachyon smears the hairpin brane by an amount whose size increases with the gradient and diverges when the mass of the open string tachyon at infinity goes to zero.

We also discussed Euclidean and Lorentzian black hole solutions. We argued that in this case α' corrections give rise to the stretched horizon of [38], *i.e.* they smear the geometry in a region outside the horizon of the black hole in which the locally measured Hawking temperature exceeds the Hagedorn temperature. When the asymptotic Hawking temperature of the black hole is small, the size of the stretched horizon is of order one in string units, but as it increases, the stretched horizon grows and eventually extends all the way to infinity when the Hawking temperature reaches the Hagedorn one.

The stretched horizon is due to a condensate of closed strings near the horizon of the black hole. In Euclidean space, this is encoded by a condensate of a tachyon wrapped around the Euclidean time direction. This condensate smears the Euclidean black hole geometry. The size of the smearing increases with the Hawking temperature of the black hole, and diverges when the mass of the wound tachyon at infinity goes to zero.

The above description of black holes with Hawking temperature of order the string scale is relevant for the string/black hole transition [38,39]. A puzzle that was raised in these papers, and was further discussed in [51,52] is the following. Near the transition, the size of the black hole, as measured by its Schwarzschild radius, is of order one in string units. However, for small string coupling, a generic string state with the same energy is very large. How could a small black hole smoothly turn into a large string?

It was proposed in [38,39,51,52] that one ingredient in the resolution of the puzzle is that string states might shrink due to the effects of gravitational self-interactions. Here we discussed another relevant effect. As the Schwarzschild radius decreases and the Hawking temperature increases, the size of the stretched horizon grows, until the Hawking temperature reaches the Hagedorn temperature, where it diverges. Thus, for a Hawking temperature slightly below the Hagedorn temperature, a classical black hole is surrounded by a very large stringy halo, and becomes harder and harder to distinguish from a generic free string state with the same energy. This picture explains why the entropies of strings and black holes must agree at the transition point.

As mentioned in section 4, the above picture is strictly valid for a black hole in a heat bath at its Hawking temperature, which is what one gets by continuation from Euclidean space. In this case, an observer at a fixed distance from the horizon perceives a thermal bath at the local Hawking temperature (4.18). When the Hawking temperature at infinity approaches the Hagedorn temperature, the properties of the system are dominated by the heat bath, and it becomes more and more difficult to distinguish it from fundamental strings at the same temperature.

When the heat bath is absent, the "white hole" modes are still in a thermal state at the local Hawking temperature, but the "past infinity" modes are in their ground state. In this situation, it might be argued that the definition (4.20) of the stretched horizon is less natural, since an observer at that distance does not perceive the temperature around him to be T_h. However, it is possible that in that case too, the definition (4.20) is meaningful: for shorter distances the black hole can not be distinguished from a fundamental string state, while for much larger distances it can.

The above discussion also provides in principle a way to compare the entropies of black holes and strings, including coefficients. If one can construct a d dimensional Euclidean Schwarzschild black hole in asymptotically flat spacetime as an exact worldsheet CFT, we expect that solution to have the following properties:

(1) The closed string tachyon with winding number one around Euclidean time at infinity should have a non-zero condensate. This condensate is normalizable when the Hawking temperature is below the Hagedorn temperature (*i.e.* the circumference of Euclidean time at infinity is larger than 4π) and non-normalizable otherwise. Thus, the Euclidean Schwarzschild solution should be non-normalizable when the Hawking temperature is above the Hagedorn temperature.

(2) At the point where the Hawking temperature of the α' corrected solution is equal to the Hagedorn temperature of strings in flat spacetime, (4.19), the black hole entropy should agree with the string one, (4.37).

For two dimensional black holes for which the exact solution is known, the above properties were verified in [45]. For d dimensional Schwarzschild black holes, we saw in section 4 that even without taking α' corrections into account, the black hole and string entropies agree to within a factor $(d-3)/(d-2)$. It would be interesting to see whether the disagreement decreases when one takes into account higher order α' corrections to the Einstein Lagrangian.

Our discussion is also related to that of [53], where it was argued that string theory at finite temperature undergoes a first order phase transition, at a temperature below the Hagedorn temperature, which is described in the Euclidean time formalism by condensation of the thermal scalar, a tachyon which winds once around Euclidean time. Physically, one would expect the transition to be to a Euclidean black hole, which as we saw is indeed likely to have a non-zero condensate of the wound tachyon, and moreover, slightly below the Hagedorn temperature, this wound tachyon is the leading deviation from flat space that one sees at infinity (since the horizon of the black hole has size of order l_s at that point). See [54] and references therein for some related comments.

The phenomena discussed in this paper have other applications as well. For example, it was shown in [55,13] that the DBI Lagrangian describing the dynamics of D-branes in the gravitational potential of other objects, such as $NS5$-branes or other D-branes, has the property that generic solutions of the equations of motion develop inhomogeneities that grow with time, and caustics. Our analysis of the hairpin brane shows that when such inhomogeneities start forming, the curved regions can become fuzzy in the same way as the vicinity of the tip of the hairpin in our discussion in section 2. This occurs since such curved branes have non-zero condensate of the open string tachyon, which is light near local minima of the D-brane shape, and its fluctuations smear the shape of the brane there. Thus, α' effects are expected to smooth out the singularities seen in the DBI approximation. Similarly, more general solutions corresponding to accelerating D-branes in string theory are expected to develop stringy smearing similar to that found in section 3.

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